Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; NELSON L. RORAY, Palmyra, N. Y., and the PROPOSER.

The general term $u_n=\frac{n+3}{n(n+1)(n+2)}.\frac{1}{3^n}$.

$$u_n=\frac{1}{2n(n+1)}.\frac{1}{3^{n-1}}-\frac{1}{2(n+1)(n+2)}.\frac{1}{3^n}.$$

$$\therefore\ u_1=\frac{1}{2.2.1}-\frac{1}{2.2.3.3},$$

$$u_2=\frac{1}{2.2.3.3}-\frac{1}{2.3.4.3^2},$$

$$u_3=\frac{1}{2.3.4.3^2}-\frac{1}{2.4.5.3^3},$$

....

$$u_n=\frac{1}{2n(n+1).3^{n-1}}-\frac{1}{2(n+1)(n+2).3^n}.$$

Adding, we get for the sum

$$S_n=\tfrac{1}{4}-\frac{1}{2(n+1)(n+2)}.\frac{1}{3^n}.$$

Also solved by *ELMER SCHUYLER*.

128. Proposed by ELMER SCHUYLER, B. Sc., Teacher of German and Mathematics, Boys' High School, Reading, Pa.

Solve $(1+x^3)(1+x^2)(1+x)=30x^2$.

I. Solution by JOHN A. VAN GROOS, B. S., Graduate Student and Assistant in Mathematics, University of Oregon, Eugene, Ore.

$$(1+x^3)(4+x^2)(1+x)=30x^3\ \ldots.(1).$$

Writing the equation in the form

$$\left(x^3+\frac{1}{x^3}\right)+\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)=28,$$

by the theory of reciprocal equations, we have

$$v_3+v_2+v_1=28,\ \text{where}\ v_3=z^3-3z,\ v_2=z^2-2,\ v_1=z=x+1/x.$$

Substituting these values, we have

$$z^3+z^2-2z-30=0.$$

By Newton's method of divisors we find that 3 is a root of this equation.

$\therefore\ (z-3)(z^2+4z+10)=0.\quad \therefore\ z=3,\ z=-2+\surd(-6),\ z=-2-\surd(-6)$.
Since $z=x+1/x$, we have

$$x=\tfrac{1}{2}(3\pm\surd5),\ x=\frac{\surd(-6)-2\pm\surd[-6-4\surd(-6)]}{2},$$

$$x=\frac{-\surd(-6)-2\pm\surd[4\surd(-6)-6]}{2},$$

for the roots of the equation.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa., and P. S. BERG, Larimore, N. D.**

Divide through by x^3 and we get

$$(x^2+1/x)(x+1/x)(1+1/x)=30,$$
$$(x^2+1/x+x+1/x^2)(x+1/x)=30.$$

Let $(x+1/x)=y$. $\therefore\ y^3+y^2-2y=30$.
Add $\frac{9}{4}y$ to both members,

$$y^3+y^2+\tfrac{1}{4}y=\tfrac{9}{4}y+30\ldots.(1).$$

y times (1) gives $y^4+y^3+\frac{1}{4}y^2=\frac{9}{4}y^2+30y$. $(y^2+\frac{1}{2}y)^2=\frac{9}{4}y^2+30y$.
Add $10(y^2+\frac{1}{2}y)$ to both members,

$$(y^2+\tfrac{1}{2}y)^2+10(y^2+\tfrac{1}{2}y)=\tfrac{9}{4}y^2+30y+10(y^2+\tfrac{1}{2}y).$$

Completing the square,

$$(y^2+\tfrac{1}{2}y)^2+10(y^2+\tfrac{1}{2}y)+25=\tfrac{49}{4}y^2+35y+25.\quad y^2+\tfrac{1}{2}y+5=\pm(\tfrac{7}{2}y+5).$$

$\therefore\ y^2-3y=0$, or $y^2+4y=-10$.
$\therefore\ y=0,\ 3,\ -2(1\pm\frac{1}{2}\surd-6)$.
$\therefore\ x+1/x=3$, or $x=\frac{1}{2}(3\pm\surd5)$.

$$x+1/x=-2(1+\tfrac{1}{2}\surd-6),\ \text{or}\ x=-\tfrac{1}{2}(2+\surd-6)\pm\tfrac{1}{2}\surd[4\surd(-6)-6].$$
$$x+1/x=-2(1-\tfrac{1}{2}\surd-6),\ \text{or}\ x=-\tfrac{1}{2}(2-\surd-6)\pm\tfrac{1}{2}\surd[-4\surd(-6)-6].$$

III. **Solution by J. M. BOORMAN, Woodmere, L. I., and the PROPOSER.**

Let $1+x^2=zx$. Then

$$[1-x+x^2][1+x^2][1+2x+x^2]=30x^3,\ \text{or}\ [zx-x][zx][zx+2x]=30x^3\ldots.(1).$$

$$z[z-1][z+2]=30\ldots.(2).$$

$\therefore\ z=3$.
Dividing (2) by $z-3$, we have $z^2+4z+10=0$, whence $z=-2\pm\surd-6$.
Since $x^2+1=zx$, therefore $x^2+1=3x$ or $x^2+1=x[-2\pm\surd-6]$.

$$\therefore x=\tfrac{1}{2}[3\pm\sqrt{5}] \text{ or } x=\tfrac{1}{2}\{-2\pm\sqrt{[-6]}\pm\sqrt{[-6\mp\sqrt{(-6)}]}\}.$$

Also solved by *JOHN M. ARNOLD*, *GEORGE D. BIRKHOFF*, *M. E. GRABER*, *J. SCHEFFER*, *L. C. WALKER*, and *H. C. WHITAKER*.

GEOMETRY.

THE PYTHAGOREAN THEOREM.

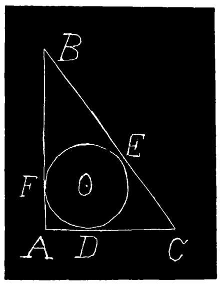
In the supposed "new proof" of the Pythagorean Theorem by Dr. Loomis, which was published last month, there appears a glaring fallacy. A number of our readers have written to us, saying that they do not understand Dr. Loomis' reasoning. The fact is, that the proof as published begs the question. That is, what is desired to be proved, is assumed. Dr. Loomis says that he had a purpose in this. He says he desired to turn the attention of thinkers to the graph submitted. The fallacy entirely escaped our notice until our attention was called to it by several of our contributors. Dr. Loomis' fallacy lies in this: If $4ar+4r^2>$ or $<2bc$, then $a^2+4ar+4r^2>$ or $<b^2+2bc+c^2$, by adding $a^2=b^2+c^2$.

Now we know that $a^2+4ar^2+4r^2=b^2+2bc+c^2$. Hence, if it is assumed that $4ar+4r^2>$ or $<2bc$, the only warranted conclusion is that $a^2<$ or $>b^2+c^2$.

Prof. B. F. Yanney says that such reasoning as employed in the proof given by Dr. Loomis would make $4ar=b^2+c^2$ or $4r^2=b^2+c^2$, or even $r^2=b^2+c^2$.

We publish the following direct proof by Professor Sawyer, which we believe will stand the test of sound reasoning. Similar direct proofs were received from W. H. Carter, D. E. Lehman, Anna L. Benschoten, and Hon. Josiah H. Drummond.

Direct proof by F. L. SAWYER, B. A., Mitchell, Ont.



Connect O with the vertices A, B, and C.

$a+2r=b+c\ldots.(1)$.

$\therefore 2a+2r=a+b+c$.

$\therefore 4ar+4r^2=2r(a+b+c)$.

Now the sum of the areas of the triangles AOB, BOC, COA=area of triangle ABC.

$\therefore \tfrac{1}{2}r(c+a+b)=\tfrac{1}{2}bc\ldots.(2)$.

$\therefore 2r(a+b+c)=2bc\ldots.(3)$.

$\therefore 4ar+4r^2=2bc$ by substituting (1) in (3).

But since $a+2r=b+c \therefore a^2+4ar+4r^2=b^2+c^2+2bc$.

$\therefore a^2=b^2+c^2$.

Problem 153, Geometry, is erroneous, it should read as follows:

If P,P', Q,Q', are the extremities of two chords of a conic section passing through the focus, A, and at right angles to each other, show that the sum of the squares of the reciprocals of AP, AP', AQ, and AQ' is constant.